Calculus for Applications, MAT 3243

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Name: _____ Pseudonym: _____

Show all work. Answers alone are not sufficient.

1. (40 pts.) Let $f(x,y,z) = (x^2+y^2+z^2)^{\frac{1}{2}}$.

   (a) Compute $f(1,0,0) - f(2,0,0)$.
   (b) Parametrize the straight line segment from $(2,0,0)$ to $(1,0,0)$.
   (c) Compute $df$.
   (d) Integrate $df$ along this segment.
   (e) Are the answers in (a) and (d) the same? Explain.

2. (40 pts.) Let $F(x,y,z) = (-x,-y,-z)$ and let $B = \{(x,y,z) \in \mathbf{R}^3 \colon x^2+y^2 \le 1, -2 \le z \le 2\}$. Note that $B$ is a solid cylinder.

   (a) Parametrize $B$.
   (b) Find the flux of $F$ through the side of the cylinder $\{(x,y,z) \in \mathbf{R}^3 \colon x^2+y^2 = 1, -2 \le z \le 2\}$.
   (c) Find the flux of $F$ through the top $\{(x,y,z) \in \mathbf{R}^3 \colon x^2+y^2 \le 1, z = 2\}$.
   (d) Find the flux of $F$ through the bottom $\{(x,y,z) \in \mathbf{R}^3 \colon x^2+y^2 \le 1, z = -2\}$.
   (e) Compute the divergence $\nabla \cdot F$.
   (f) Integrate $\nabla \cdot F$ over $B$.
   (g) Are the answers in (b)+(c)-(d) and (f) the same? Explain.

3. (40 pts.) (40 pts.) Let $\omega = -y\,dx + x\,dy$.

   (a) Integrate $\omega$ over the unit circle $\{(x,y) \in \mathbf{R}^2 \colon x^2+y^2 = 1\}$.
   (b) Compute $d\omega$.
   (c) Integrate the flux of $d\omega$ through the unit disc $\{(x,y) \in \mathbf{R}^2 \colon x^2+y^2 \le 1\}$.
   (d) Are the answers in (b) and (c) the same? Explain.

| 1a | 1b | 1c | 1d | 2a | 2b | 3a | 3b | total (160) |
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