Calculus for Applications, MAT 3243

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Name: ______________________ Pseudonym: ______________________

Show all work. Answers alone are not sufficient.

1. (24 pts.) Parametrize the following regions. Specify the ranges for the parameters.

   (a) The straight line segment from $(2,-1,3)$ to $(1,0,-2)$.
   (b) The plane in $\mathbf{R}^3$ containing $(0,0,1)$, $(1,0,0)$, $(0,1,0)$.
   (c) The unit sphere $\{(x,y,z)\in\mathbf{R}^3\colon x^2+y^2+z^2=1\}$.
   (d) The southern hemisphere of the unit sphere $\{(x,y,z)\in\mathbf{R}^3\colon x^2+y^2+z^2=1, z\le 0\}$.
   (e) The part of the unit sphere contained in the positive orthant: $\{(x,y,z)\in\mathbf{R}^3\colon x^2+y^2+z^2=1, x\ge 0, y\ge 0, z\ge 0\}$.
   (f) The unit ball $\{(x,y,z)\in\mathbf{R}^3\colon x^2+y^2+z^2\le 1\}$.
   (g) The ball of diameter 4 centered at $(5,3,-1)$.
   (h) The graph of $z=f(x,y)$ in $\mathbf{R}^3$, where $f\colon\mathbf{R}^2\to\mathbf{R}$ is a continuous function.

2. (32 pts.) Evaluate the following integrals

   (a) $\int x\,dx+y\,dy+z\,dz$ along the segment $\{(2-t,t,-1+t)\colon 0\le t\le 1\}$.
   (b) $\iint x\,dy\,dz+y\,dz\,dx+z\,dx\,dy$ through the cylinder $\{(\cos\theta,\sin\theta,z)\colon 0\le\theta\le 2\pi, -2\le z\le 2\}$.
   (c) $\iint x\,dy\,dz+y\,dz\,dx+z\,dx\,dy$ through the disk $\{(\rho\cos\theta,\rho\sin\theta,2)\colon 0\le\rho\le 1, 0\le\theta\le 2\pi\}$.
   (d) $\iiint\left(x^2+y^2\right)dx\,dy\,dz$ over the solid cylinder $\{(\rho\cos\theta,\rho\sin\theta,z)\colon 0\le\rho\le 1, 0\le\theta\le 2\pi, -2\le z\le 2\}$.

3. (16 pts.) True/false questions. Circle your choice. Justification is not necessary.
   In this question all functions are differentiable on $\mathbf{R}^3$.
   Lowercase functions are $\mathbf{R}^3\to\mathbf{R}$ and uppercase functions are $\mathbf{R}^3\to\mathbf{R}^3$.

T F (a) The integral of df around any circle is 0.
T F (b) The flux of the curl $\nabla\times F$ through a sphere is 0.
T F (c) The integral of divergence $\nabla\cdot F$ over a ball is 0.
T F (d) If $\nabla\times F=0$, then F is a gradient, i.e. $F=\nabla f$ for some f.
T F (e) If $df=0$, then $f=0$.
T F (f) $\nabla\times(\nabla f)=0$
T F (g) $\nabla(\nabla\cdot F)=0$
T F (h) $\nabla\cdot(\nabla\times F)=0$

1	2a	2b	2c	2d	3	total (72)	%