Calculus for Applications, MAT 3243
Midterm, October 18, 1995
Instructor: D. Gokhman

Name: _____

Show all work. Answers alone are not sufficient.

1. (30 pts.) Find parametric formulas $(x(t), y(t))$ for the following curves in $\mathbf{R}^2$ (remember to specify the range of the parameter t):
   (a) The line segment from $(-1, 2)$ to $(5, -3)$.
   (b) The circle of radius 5 centered at $(-2, -1)$.
   (c) The graph of $y = x^3$.
2. (40 pts.) Let $f\colon \mathbf{R}^2 \to \mathbf{R}$ be the projection to the diagonal, i.e. $f(x, y) = (x+y)/\sqrt{2}$, and let $g\colon \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation from polar to cartesian coordinates, i.e. $g(r, \theta) = (r\cos\theta, r\sin\theta)$.
   (a) Find $h = f \circ g$ — the composition of f and g: $h(r, \theta) = f(g(r, \theta))$.
   (b) Find the derivative matrices $D(f)$, $D(g)$, $D(h)$.
3. (60 pts.) Let $f(x, y) = 4x^2 + y^2$.
   (a) In the x-y plane sketch the level curves of the graph $z = f(x, y)$ at heights $c = 0, 1, 4$.
   (b) Find ∇f at $p = (-1/2, 0)$ and sketch it. Is it perpendicular to the level curve passing through p?
   (c) Find the directional derivative of f along $(1, -1)$ at p.
   (d) Extra credit: sketch the graph of $z = f(x, y)$.
4. (40 pts.) Integrate $x\,dx - y\,dy$ around the unit circle $(\cos(t), \sin(t))$, $0 \le t < 2\pi$

1	2	3	4	total (170)