Calculus for Applications, MAT 3243 (Extra)
Final, Fall, 1995
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Name: _____ Pseudonym: _____

Show all work. Answers alone are not sufficient.

1. (40 pts.) Let $\omega = 3x^2y\,dx + (x^3 + 3y^2e^z)\,dy + y^3e^z\,dz$

   (a) Show that $d\omega = 0$ (i.e. $\operatorname{curl}(3x^2y, x^3 + 3y^2e^z, y^3e^z) = 0$).

   (b) Find $f(x, y, z)$ such that $df = \omega$ (i.e. $\operatorname{grad} f = (3x^2y, x^3 + 3y^2e^z, y^3e^z)$).
   [Hint: guess/check or integrate along a straight line $(0, 0, 0) \to (x, y, z)$]

   (c) Evaluate $\int_{(1,2,-2)}^{(0,-1,2)} \omega = \int_{(1,2,-2)}^{(0,-1,2)} 3x^2y\,dx + \left(x^3 + 3y^2e^z\right)\,dy + y^3e^z\,dz$
   [Hint: you may use part (b) and the Fundamental Theorem of Calculus.]

   (d) What is the integral of ω around a closed curve? Explain.

2. (40 pts.) Let $F(x, y, z) = (2x, 2y, -5z)$. Find the flux $\int F \cdot dS$ through:

   (a) the cylinder $\Phi(\theta, z) = (\cos\theta, \sin\theta, z)$, $0 \le \theta < 2\pi$, $-3 \le z \le 3$

   (b) the disc $\Phi(\rho, \theta) = (\rho\cos\theta, \rho\sin\theta, 6)$, $0 \le \rho \le 3$, $0 \le \theta < 2\pi$

3. (40 pts.) Evaluate the following integrals:

   (a) $\int_S 3x^2y^4z^2\,dx + 4x^3y^3z^2\,dy + 2x^3y^4z\,dz$,

   where S is the unit circle in the x-y plane ($S = \{(x, y, z)\colon x^2 + y^2 = 1, z = 0\}$).

   (b) $\int_D \cos(y)z^2\,dy\,dz - \left(z^4 + 1\right)^x\,dz\,dx + e^{xy}\,dx\,dy$,

   where D is the unit sphere ($D = \{(x, y, z)\colon x^2 + y^2 + z^2 = 1\}$).

   [Hint: S and D are boundaries.]

1a	1b	1c	1d	2a	2b	3a	3b	total (120)