Calculus for Applications, MAT 3243
Midterm, October 19, 1994
Instructor: D. Gokhman

Name: ______________________________

1. (40 pts.) Find matrices that represent the following linear maps $f : \mathbf{R}^3 \to \mathbf{R}^3$ with respect to the standard basis for $\mathbf{R}^3$:
   (a) projection to the x-z plane,
   (b) reflection with respect to the x-z plane,
   (c) rotation by π around the y axis clockwise if you look from the positive y direction,
   (d) projection to the line $\ell(t) = t(1, 1, 1)$.

2. (40 pts.) Find the determinants and inverses of the following matrices:

$$\text{(a)} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \qquad \text{(b)} \begin{pmatrix} 3 & 2 & 1 \\ 0 & 3 & 2 \\ 0 & 0 & 3 \end{pmatrix}$$

3. (30 pts.) Find parametric formulas for the following curves in $\mathbf{R}^2$:
   (a) The line through $(-1, 2)$ and $(5, -3)$.
   (b) The circle of radius 5 centered at $(-1, -1)$.
   (c) The parabola $x = y^2$.

4. (40 pts.) Suppose that the position of a particle in $\mathbf{R}^2$ as a function of time $t \geq 0$ is given by $r(t) = (t \cos(2\pi t), t \sin(2\pi t))$.
   (a) Sketch the trajectory of the particle.
   (b) Find the velocity as a function of t.
   (c) Find the speed as a function of t (simplify!).
   (d) Find a parametric formula for the line tangent to the trajectory at the point $r(1)$.

A useful formula:

Projection of u to the line through the origin spanned by $v \neq 0$ is $\dfrac{u \cdot v}{||v||^2} v$.

1	2	3	4	total (150)