Midterm 3 / 2015.4.24 / MAT 3233.001 / Modern Algebra

Name: _____

Please show all work and justify your answers.

1. Show that $n + n^3 + n^5$ is always divisible by 3.
2. Suppose $\gcd(a, m) = 1$. Given a pair of multiplicative inverses e and d in $U(\varphi(m))$, prove that $(a^e)^d \equiv a \bmod m$.
3. Find all cosets of the subgroup $\langle 11 \rangle < U(45)$. For each coset find its order as an element of the quotient group $U(45)/\langle 11 \rangle$.
4. Find all solutions to the system of congruences $x \equiv 2 \bmod 3$, $x \equiv 1 \bmod 4$, $x \equiv 3 \bmod 5$.

1	2	3	4	total (40)