

Name: \_\_\_\_\_

Please show all work.

1. Partition the symmetric group  $S_3$  by left cosets of the cyclic subgroup  $\langle(2, 3)\rangle$ . Do the same with right cosets.
2. Suppose  $G, G'$  are commutative multiplicative groups and  $\varphi : G \rightarrow G'$  is a surjective homomorphism. For  $y$  in  $G'$  express its fibre  $\varphi^{-1}(y) = \{x \in G : \varphi(x) = y\}$  as a coset of  $\ker \varphi$ .
3. Find the solution set for the system of congruences

$$35x \equiv 15 \pmod{50}$$

$$x \equiv -2 \pmod{30}$$

4. Use Euclid's algorithm for the polynomial ring  $\mathbf{R}[x]$  to find the greatest common divisor and the Bézout coefficients for  $x^2 + 3x + 2$  and  $x^4 + x^3 + 3x + 3$ .
5. Suppose  $a$  is a real number and  $\varphi : \mathbf{R}[x] \rightarrow \mathbf{R}$  is the evaluation map  $\varphi(p(x)) = p(a)$ . Prove that  $\varphi$  is a ring homomorphism. What are its kernel and image?

1	2	3	4	5	total (50)	%

Prelim. course grade:                      %