

Name: _____

Please show all work.

1. Prove by induction that $n! \leq n^n$ for all natural numbers n .
2. Use Euclid's algorithm to find the gcd and the Bezout coefficients for 58 and 44.
3. Suppose a, r, m are natural numbers with $a \equiv r \pmod{m}$. Prove that $\gcd(a, m) = \gcd(r, m)$.
Hint: express r in terms of a and m and show that the two gcd's divide one another.
4. Find all solutions modulo 33 of the linear congruence $15x \equiv 21 \pmod{33}$.
5. Prove that any nonzero element in a finite commutative ring with unity is either a unit or a zero divisor, but not both.

Hint: apply the pigeonhole principle to the sequence of positive integer powers of the element.

1	2	3	4	5	total (50)	%

Prelim. course grade: %