Midterm 1 / October 20, 2010 / MAT 3233.001 / Modern Algebra

Name: ____

Please show all work.

1. Use induction to show that for $n \geq 1$ the partial sum

$$1 + 7 + 13 + \ldots + (6n - 5) = \sum_{k=1}^{n} (6k - 5)$$

   can be expressed in closed form by $3n^2 - 2n$.

2. Use Euclid's algorithm to find $(75, 27)$ and $s, t \in \mathbf{Z}$ such that $(75, 27) = 75s + 27t$.

3. Find all solutions of the linear congruences

$$\text{(a) } 3x \equiv 5 \bmod 13 \qquad \text{(b) } 5x \equiv 15 \bmod 20$$

4. Compute 3^{42} modulo 7 by repeated squaring and reduction. Show work.

5. Suppose R is a commutative ring (with unity) and let U be the set of all units in R.
   (a) Prove that U is a multiplicative group.
   (b) Prove that U cannot contain zero divisors.
   (c) Describe U for the rings $\mathbf{Z}$, $\mathbf{Z}_m$, and $\mathbf{C}$.

1	2	3	4	5	total (50)	%

Prelim. course grade: %