Complex Variables / MAT 3223.001
Midterm 2 / 2002.4.24 / Instructor: D. Gokhman

Name: ______________________________

1. (10 pts.) Find all roots of $ze^{4iz} + 2z + ze^{-4iz}$ inside the unit disc and determine their multiplicity.

2. (10 pts.) Show that all 4 roots of z^4+z^2-z-5 are located in the annulus $\{z : 1 \leq z \leq 2\}$.

3. (20 pts.) Evaluate the following integrals along the given paths (sketch):

   (a) $\displaystyle\int_\gamma \frac{dz}{z^3 + 2z^2}$, where γ is the unit circle

   (b) $\displaystyle\int_\gamma \overline{z}\, dz$, where γ is the straight line segment from i to -1

   (c) $\displaystyle\int_\gamma \overline{z}\, dz$, where $\gamma = \{z\colon\ |z - 1 + i| = 1\}$

   (d) $\displaystyle\int_\gamma \frac{z\, dz}{z^2 + i}$, where $\gamma = \{z\colon\ |z - 1 + i| = 1\}$

4. (10 pts.) Let $I(r) = \displaystyle\int_\gamma \frac{1}{z^2 + 1}\, dz$, where $\gamma = \{re^{it}\colon 0 \leq t \leq \pi\}$ with $r > 1$.

   Estimate $|I(r)|$ and show that $I(r) \to 0$ as $r \to \infty$.

1	2	3	4	total (50)	%