Complex Variables / MAT 3223.001
Midterm 1 / Mar. 1, 2000 / Instructor: D. Gokhman

1. (10 pts.) Sketch the locus of each of the following equations in the complex plane.

$$\text{(a) } |z-1+i| = |z+1+i| \qquad \text{(b) } \arg\left(\frac{z-i}{z-1}\right) = \pi/2$$

2. (10 pts.) Let $f(z) = e^z$. Parametrize and sketch each of the following lines.

   Then find and sketch their image under f.

$$\text{(a) } \operatorname{Re} z = 1 \qquad \text{(b) } \operatorname{Im} z = -1$$

3. (20 pts.) Find the Taylor series expansion of each of the following functions at the given center c. In each case determine and sketch the disc of convergence.

$$\text{(a) } \frac{1}{1+3z}, c=0 \qquad \text{(b) } \frac{1}{1-z}, c=2 \qquad \text{(c) } \log(z), c=1 \qquad \text{(d) } \frac{z^5}{e^{z^2}}, c=0$$

4. (20 pts.) Consider $f: \mathbf{C} \to \mathbf{C}$.

   (a) Suppose f is a global expansion by a factor of 12 composed with a clockwise rotation by $\pi/4$. Write down a formula for $f(z)$.

   (b) Suppose $f(z) = 2i\log(z)$. Describe geometrically the local behaviour of f in the vicinity of $z = 1+i$.

5. (20 pts.) Find all points in the complex plane, where each of the following functions of $z = x+iy$ is complex differentiable.

   In each case, where the function is complex differentiable, find the derivative.

$$\text{(a) } x^2 + iy^2 \qquad \text{(b) } x^2 + y^2 + 2ixy$$

1	2	3	4	5	total (80)	%