Complex Variables / MAT 3223.001
Final exam / May 11, 2000 / Instructor: D. Gokhman

Name: ____________________ Pseudonym: ____________________

1. (10 pts.) Sketch the locus of each of the following equations in the complex plane.

$$\text{(a) } |z-i| = |z+1| \qquad \text{(b) } \arg(z-i) = -\pi/4$$

2. (10 pts.) Let $f(z) = e^z$. Parametrize and sketch each of the following lines.
Then find and sketch their image under f.

$$\text{(a) } \operatorname{Re} z = -2 \qquad \text{(b) } \operatorname{Im} z = 2$$

3. (20 pts.) Find the Taylor series expansion of each of the following functions at the given center c. In each case determine and sketch the disc of convergence.

$$\text{(a) } \frac{1}{3+z}, c=0 \qquad \text{(b) } \frac{1}{3+z}, c=1 \qquad \text{(c) } \log(z), c=2 \qquad \text{(d) } \frac{z^7}{e^{z^3}}, c=0$$

4. (10 pts.) Consider $f\colon \mathbf{C} \to \mathbf{C}$.
   (a) Suppose $f(z) = -i\log(z)$. Describe geometrically the local behaviour of f in the vicinity of $z = 1-i$.
   (b) Suppose f is a global expansion by a factor of $1/2$ composed with a clockwise rotation by $\pi/2$. Write down a formula for $f(z)$.

5. (10 pts.) Find all points in the complex plane, where each of the following functions of $z = x+iy$ is complex differentiable.
In each case, at the points where the function is complex differentiable, find the derivative.

$$\text{(a) } \frac{1}{x-iy} \qquad \text{(b) } e^{-y}(\cos x + i\sin x)$$

6. (10 pts.) Find all roots of f in the unit disc and determine their multiplicity.

$$\text{(a) } f(z) = \cos(4z) + 1 \qquad \text{(b) } f(z) = e^{4z} + i$$

7. (10 pts.) Integrate $f(z)\,dz$ along the straight line segment from $i-1$ to 1.

$$\text{(a) } f(z) = \operatorname{Im} z \qquad \text{(b) } f(z) = \overline{z}z$$

8. (20 pts.) Integrate around the unit circle once counterclockwise.

$$\text{(a) } \int \frac{dz}{i-2z} \qquad \text{(b) } \int \frac{dz}{z^3-2z^2} \qquad \text{(c) } \int \frac{\cos(z^2)}{z^7}\,dz \qquad \text{(d) } \int \frac{dz}{z\cos z - z}$$

9. (10 pts.) Show that all three roots of $p(z) = z^3 - z - 4$ lie in the annulus $1 < |z| < 2$.

1	2	3	4	5	6	7	8	9	total (110)	%