

Name: _____ Pseudonym: _____

Please show all work and box the answers, where appropriate.

1. (20 pts.) Find all solutions of $z^3 = -8$, express them in polar and cartesian form, and sketch them.
2. (10 pts.) Find all values of $\log i$ and sketch a few of them.
3. (30 pts.) Find the Maclaurin series expansion of each of the following functions and determine its radius of convergence.

(a) $\frac{1}{1-3z}$

(b) $\frac{1}{i-z}$

4. (20 pts.) Find all branch points of $f(z) = \sqrt{1+z^4}$. What is the smallest number of branch cuts needed to make f single valued? Sketch the branch points and an example of branch cuts as above. What is the radius of convergence of the Taylor series expansion of f at 1?
5. (20 pts.) Find all points in the complex plane, where each of the following functions of $z = x + iy$ is analytic? complex differentiable?

(a) $e^y(\sin x + i \cos x)$ (b) $x^2 + y^2$

6. (40 pts.) Evaluate integrals (a) and (b) around the unit circle, and integrals (c) and (d) along the straight line segment from $-i$ to $1 + i$.

(a) $\int \frac{dz}{z^2 - 3z}$ (b) $\int \frac{dz}{z^4 - 3z^3}$ (c) $\int |z|^2 dz$ (d) $\int \bar{z} dz$

7. (20 pts.) Let $p(z) = z^5 + 4z^2 - 1$.

- (a) Show that all five zeros of p lie in the disc $|z| < 2$.
- (b) Show that exactly two zeros of p lie in the unit disc.

1	2	3	4	5	6	7	total (160)	%