University of Texas at San Antonio

Complex Variables, MAT 3223
Exam №2, 4/13/92
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Name: ______________________________

You may use any theorem that has a name attached to it.

1. (30 pts.) For the following functions $f(z)$ and contours Γ, sketch Γ and calculate the contour integral of $f(z)$ along Γ:
   (a) $f(z) = (\overline{z})^2$, Γ is part of $y = x^2$ from $(0,0)$ to $(1,1)$.
   (b) $f(z) = \operatorname{Im} z$, Γ is $|z| = 1$ (clockwise).
2. (28 pts.) For the following functions $f(z)$ and closed contours Γ, sketch Γ and use the Cauchy Integral Formula to calculate the contour integral of $f(z)$ counterclockwise along Γ:
   (a) $f(z) = z\left(z^2 - 1\right)^{-1}$, Γ is $|z - \pi| = 1$.
   (b) $f(z) = z^{-3} \cos z$, Γ is $|z + 2i| = 1$.
3. (42 pts.) Prove the following propositions:
   (a) If the contour integral of $f(z)$ along any closed curve in $\mathbf{C}$ equals zero, then any integral of $f(z)$ is path independent.
   (b) If $f(z)$ is entire, then so is $f'(z)$.
   (c) If $p(z)$ is a polynomial of degree ≥ 1, then $f(z) = p(z)^{-1}$ is not entire. Where is $f(z)$ differentiable?