University of Texas at San Antonio

Complex Variables, MAT 3223
Exam $\mathcal{N}^{\underline{o}}1$, 3/4/92
Instructor: D. Gokhman

Name: ____

1. (25 pts.) For the following functions $f: D \to \mathbf{C}$ find the largest domain $D \subseteq \mathbf{C}$ where f is differentiable. Verify the Cauchy-Riemann equations for f on D in part (a) only.
   (a) $f(z) = 1/(z^2 + 1)$
   (b) $f(z) = 1/(z^6 - 5iz)$.
2. (20 pts.) For each of the following sets $S \in \mathbf{C}$
   (a) $S = \{z \in \mathbf{C}\colon\ |z| < 1, |\operatorname{Re} z| \neq |\operatorname{Im} z|\}$,
   (b) $S = \{z \in \mathbf{C}\colon\ |z - 1| < |z + 1|\}$,

   sketch S. Determine whether S is
   (i) open,
   (ii) a domain,
   (iii) bounded,

   and find
   (iv) the closure of S,
   (v) the boundary of S.
3. (30 pts.) Prove the following statements:
   (a) If $z = x + iy \in \mathbf{C}$, then $|x| + |y| \leq \sqrt{2}\,|z|$.
   (b) If $z_1, z_2 \in \mathbf{C}$, then $|z_1 z_2| = |z_1|\,|z_2|$.
4. (25 pts.) Suppose $D = \{z \in \mathbf{C}\colon\ |z| < 1\}$. Classify all differentiable functions $f : D \to \mathbf{C}$ such that $|f| = \text{const}$ on D. (Hint: Write $f(z)$ in polar form and use the Cauchy-Riemann equations to show that $f = \text{const}$ on D.)