University of Texas at San Antonio

Complex Variables, MAT 3223
Final Exam, 5/7/92
Instructor: D. Gokhman

Name: ________________________________

1. (48 pts.) Find and sketch all z such that:
   (a) $e^z = 0$.
   (b) $\operatorname{Re}(e^z) = 0$.
   (c) The function $f(\zeta) = \zeta \operatorname{Re} \zeta$ is differentiable at z.

2. (52 pts.) CONTOUR INTEGRATION.
   Evaluate the following contour integrals and sketch the contours.
   (a) Evaluate the contour integral of $\overline{z}$ along the straight line from 1 to $2i$.
   (b) Let $H = \{z\colon \operatorname{Im} z > 0\}$. Let $A = (B[0,2] - \overline{B[0,1]}) \cap H$. Evaluate the contour integral of $z/\overline{z}$ around ∂A (positively oriented).

3. (52 pts.) CAUCHY INTEGRATION THEORY.
   Evaluate the following contour integrals:

$$\text{(a)} \quad \int_{\partial B[0,4]} \frac{e^z}{(z^2+\pi^2)}\,dz. \qquad \text{(b)} \quad \int_\Gamma \frac{\sin(2z)}{(z+\pi)^2}\,dz,$$

   where Γ is the square with corners $5 \pm 5i, -5 \pm 5i$ traversed counterclockwise. Sketch the contours and all singularities.

4. (48 pts.) SERIES.
   Find Laurent series for $f(z) = z(z^2+3z+2)^{-1}$ valid in the following regions:

$$\text{(a) } B[0,1]. \quad \text{(b) } B[0,2] - \overline{B[0,1]}. \quad \text{(c) } \mathbf{C} - \overline{B[0,2]}.$$

   Sketch the regions.
   Hint: partial fractions.