

Name: \_\_\_\_\_

Please show all work. If you use a theorem, name it or state it.

1. Suppose  $\forall n \ x_n \in \mathbf{R}$  with  $|x_n| < 1/n$ . Prove that the sequence  $(x_n)$  is Cauchy directly from the definition. What is the limit of  $(x_n)$ ?
2. Suppose  $\forall n \ a_n > 0$  and the series  $\sum a_n$  converges.
  - (a) Prove that  $\sum a_n^2$  converges.
  - (b) Show by example that  $\sum \sqrt{a_n}$  need not converge.
3. Use the definition of limit to prove that  $x^2 + x + 1 \rightarrow 7$  as  $x \rightarrow 2$ .
4. Find the limit of  $\frac{x}{x+1}$  as  $x \rightarrow -1^+$ . Prove your assertion.

1	2	3	4	total (40)