Final exam / 2017.5.9 / MAT 3213.002 / Foundations of Analysis

Name: ______________________________

Please show all work. If you use a theorem, name it or state it.

1. Suppose C and D are Dedekind cuts. Prove that their intersection $C \cap D$ is a Dedekind cut. Give a concrete example of a sequence of Dedekind cuts (D_n) whose intersection is not a Dedekind cut.
2. Find all real x such that $3 < |x-2| + |x+1| < 7$.
3. Suppose A, B are nonempty bounded subsets of $\mathbf{R}$ that are not disjoint. Prove that $\inf(A \cap B) \geq \min\{\inf A, \inf B\}$. Give a concrete example where the inequality is strict.
4. Suppose (x_n) is sequence in $\mathbf{R}$ that is not bounded. Prove that (x_n) has a subsequence convergent to $+\infty$ or a subsequence convergent to $-\infty$.
5. Find lim sup and lim inf of the sequence $x_n = (-1)^n - \frac{1}{n}$. Prove your assertion for lim inf.
6. Suppose (x_n) is a bounded sequence and $\limsup x_n$ and $\liminf x_n$ belong to an open interval (a, b). Prove that $\exists k \in \mathbf{N}\ \forall n \in \mathbf{N}\ n \geq k \Rightarrow x_n \in (a, b)$.
7. Prove that every convergent sequence in $\mathbf{R}$ is Cauchy.
8. Suppose $\sum x_n$ is convergent. Prove that the sequence (x_n) converges to 0.

1	2	3	4	5	6	7	8	total (80)