Foundations of Analysis / MAT 3213.001

Midterm 1 / February 16, 1998 / Instructor: D. Gokhman

Name: ______________________________

1. (10 pts.) Suppose $\delta > 0$. Prove that $|x-1| < \delta \Rightarrow |x^2-1| < \delta(\delta+2)$.

2. (10 pts.) Solve $2\,|x| \geq |x-1|$ for x.

3. (10 pts.) Suppose A is a nonempty bounded subset of $\mathbf{R}$ such that $\inf A = \sup A$. Prove that A has exactly one element.

4. (10 pts.) Suppose A and B are bounded subsets of $\mathbf{R}$ with $A \cap B \neq \emptyset$. Prove that $A \cap B$ is bounded below and $\inf(A \cap B) \geq \max\{\inf A, \inf B\}$.

5. (10 pts.) Let $\mathbf{R}^+ = \{x \in \mathbf{R}\colon x > 0\}$. Suppose A and B are nonempty bounded subsets of $\mathbf{R}^+$. Let $C = \{x\colon \exists a \in A, b \in B \text{ such that } x = ab\}$. Prove that $\sup C = \sup A \sup B$.

   Extra credit: What can you say about $\sup C$, if we remove the restriction that the elements of A and B are positive?

1	2	3	4	5	total (50)	%