Foundations of Analysis / MAT 3213.001

Final / May 5, 1998 / Instructor: D. Gokhman

Name: ____________________ Pseudonym: ____________________

1. (10 pts.) Suppose $\delta > 0$. Prove that $|x-2| < \delta \Rightarrow |x^2-4| < \delta(\delta+4)$.
2. (10 pts.) Find all $x \in \mathbf{R}$ such that $|2x| \le |x-2|$.
3. (20 pts.) Suppose $f\colon A \to B$ and $g\colon B \to C$.
   (a) Prove that if f and g are 1-1, then $g \circ f$ is 1-1.
   (b) Prove that if f is onto and $g \circ f$ is 1-1, then g is 1-1.
4. (20 pts.) Suppose A and B are nonempty bounded subsets of $\mathbf{R}$. Prove that $A \cup B$ is bounded below and $\inf(A \cup B) = \min\{\inf A, \inf B\}$.
5. (20 pts.) Suppose A is a nonempty bounded subset of $\mathbf{R}$. Prove that $\inf A \in \partial A$ by showing that $\inf A \in \overline{A}$, but $\inf A \notin A^\circ$.
6. (20 pts.) Suppose A and B are closed subsets of $\mathbf{R}$. Determine whether each of the following statements is true and prove the statement or give a counterexample.
   (a) $(A \cap B)^\circ = A^\circ \cap B^\circ$
   (b) $(A \cup B)^\circ = A^\circ \cup B^\circ$
7. (20 pts.) Let $A = \left\{1/n\colon n \in \mathbf{Z}^+\right\}$. Determine $\sup A$, $\inf A$, $\max A$, $\min A$, A°, $\mathrm{L}(A)$ and $\overline{A}$. Is A closed in $\mathbf{R}$? Is A open in $\mathbf{R}$?
8. (20 pts.) Suppose A is a nonempty subset of $\mathbf{R}$, which is bounded below, but does not have a minimum. Prove that $\mathrm{L}(A) \neq \emptyset$.

Have a great summer!

1	2	3	4	5	6	7	8	total (140)	%