

Name: \_\_\_\_\_

Please show all work.

1. Let  $S = \left\{ \frac{n}{n+1} : n \in \mathbf{N} \right\} \subseteq \mathbf{R}$

Does  $S$  have a sup? inf? max? min? If so, find them. Prove your assertions.

2. Determine whether each of the following relations  $F$  on  $\mathbf{R}$  is a function. Explain.

(a)  $F = \{[x, y] \in \mathbf{R} \times \mathbf{R} : x - y = 1\}$

(b)  $F = \{[x, y] \in \mathbf{R} \times \mathbf{R} : x^2 + (y + 1)^2 = 1\}$

3. Let  $F: \mathbf{R} \times \mathbf{R} \rightarrow \mathbf{R}$  be the function defined by  $F(x, y) = x - y$

(a) Prove that  $F$  is surjective.

(b) Sketch the inverse images  $F^{-1}(\{0\})$  and  $F^{-1}(\{1\})$  on the same graph.

4. Define a function  $F: \mathbf{R} \rightarrow \mathbf{R}$  by  $F(x) = \begin{cases} x + 1 & \text{for } x < 0 \\ -2x & \text{for } x \geq 0 \end{cases}$

(a) Sketch the graph of  $F$  and explain why  $F$  is neither injective nor surjective.

(b) Find forward and inverse images of intervals:  $F([-1, 1])$ ,  $F^{-1}([0, \infty))$

5. Prove that  $F: (1, 2] \rightarrow (-\infty, 0]$  defined by  $F(x) = \frac{x - 2}{x - 1}$  is bijective.

1	2	3	4	5	total (50)