Midterm 2 / 2023.11.1 / MAT 3013.001 / Foundations of Mathematics

Name: ______________________________

Show all work.

1. Prove by induction that $\displaystyle\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$ for $n = 1, 2, \ldots$

2. Prove by induction that $3^n \geq 1 + 2^n$ for $n = 1, 2, \ldots$

3. Let $A = \{1, 2, 3\}$ and let $R = \{[1, 3], [2, 2], [3, 1]\}$ be a relation on A. Find $R \circ R$ and $R \circ R \circ R$ and sketch a digraph for each of the relations $R, R \circ R, R \circ R \circ R$

4. Define a relation R on $\mathbf{R} \times \mathbf{R}$ by $[x, y]R[r, s] \Leftrightarrow x + y = r + s$. Prove that R is an equivalence relation. On the same set of axes sketch the equivalence class of $[1, 2]$ and the equivalence class of $[1, 3]$

5. Explain why the set of all even integers $2\mathbf{Z}$ and the set of all odd integers $1 + 2\mathbf{Z}$ form a partition of $\mathbf{Z}$. Describe the equivalence relation on $\mathbf{Z}$ whose quotient set is the above partition $\{2\mathbf{Z}, 1 + 2\mathbf{Z}\}$

1	2	3	4	5	total (50)