Name: _____

Please show all work.

1. Let $S = \left\{ \frac{1}{n} : n \in \mathbf{N} \right\} \subseteq \mathbf{R}$

   Does S have a sup? inf? max? min? If so, find them. Prove your assertions.

2. Determine whether each of the following relations F on $\mathbf{R}$ is a function. Explain.

   (a) $F = \{[x, y] \in \mathbf{R} \times \mathbf{R} \colon x + y = 2\}$

   (b) $F = \{[x, y] \in \mathbf{R} \times \mathbf{R} \colon x^2 + y^2 = 4\}$

3. Let $F \colon \mathbf{R} \times \mathbf{R} \to \mathbf{R}$ be the function defined by $F(x, y) = x + y$

   (a) Prove that F is surjective.

   (b) Sketch the inverse images $F^{-1}(\{0\})$ and $F^{-1}(\{1\})$ on the same graph.

4. Define a function $F \colon \mathbf{R} \to \mathbf{R}$ by $F(x) = \begin{cases} x + 1 & \text{for } x < 0 \\ -x & \text{for } x \geq 0 \end{cases}$

   (a) Sketch the graph of F and explain why F is neither injective nor surjective.

   (b) Find forward and inverse images of intervals: $F([-1, 1])$, $F^{-1}([0, \infty))$

5. Prove that $F \colon [1, 2) \to (-\infty, 0]$ defined by $F(x) = \dfrac{x - 1}{x - 2}$ is bijective.

1	2	3	4	5	total (50)