

Name: _____

Please show all work.

1. Let $S = \left\{ \frac{n}{n+1} : n \in \mathbf{N} \right\} \subseteq \mathbf{R}$

Does S have a sup? inf? max? min? If so, find them. Prove your assertions.

2. Determine whether each of the following relations $S: \mathbf{R} \rightarrow \mathbf{R}$ is a function. Prove your assertions.

(a) $S = \{[x, y] \in \mathbf{R}^2: (x - 1)^2 + y^2 = 4\}$

(b) $S = \{[x, y] \in \mathbf{R}^2: |y| < 1\}$

3. Define $f: \mathbf{R}^2 \rightarrow \mathbf{R}$ by $f(x, y) = x + 2y$

(a) Prove that f is onto.

(b) Sketch the fibers $f^{-1}(\{-1\}), f^{-1}(\{0\}), f^{-1}(\{2\})$ on the same graph.

4. Define $f: \mathbf{R} \rightarrow \mathbf{R}$ by $f(x) = \begin{cases} -2x & \text{for } x < 0 \\ x - 2 & \text{for } x \geq 0 \end{cases}$

(a) Prove that f is not onto.

(b) Find the following images and preimages: $f([-1, 1]), f^{-1}([2, \infty))$

5. With f as in preceding problem, give concrete examples of subsets $E, D \subseteq \mathbf{R}$ such that $D \neq f^{-1}(f(D))$ and $E \neq f(f^{-1}(E))$

1	2	3	4	5	total (50)