

Name: _____

Please show all work.

1. Give two different proofs that the sum of all binomial coefficients $\binom{n}{k}$ for a fixed n is 2^n . One using the Binomial theorem and one not.
2. Prove that the following relations R on a set X are equivalence relations. Sketch several equivalence classes (use different colors for each class). Can you identify the quotient sets X/R ?
 - (a) $X = \mathbf{N} \times \mathbf{N}$. $[m, n]R[k, l] \Leftrightarrow m + l = k + n$.
 - (b) $X = \mathbf{R}$. $xRy \Leftrightarrow x - y \in \mathbf{Z}$.
3. Let $S = \{x \in \mathbf{Q}: \exists n \in \mathbf{N} x = 1/n\}$ Find $\max S$ and $\min S$ if they exist. Same for $\sup S$ and $\inf S$. Prove your assertions.
4. Suppose A is a set and $B_k \subseteq A$ for $k \in K$, where K is a nonempty indexing set. Let $S = \{B_k: k \in K\} \subseteq \mathcal{P}(A)$. Show that for the partial order \subseteq on $\mathcal{P}(A)$ we have $\sup S = \cup S$ and $\inf S = \cap S$.

1	2	3	4	total (40)