Name: ____________________

Please show all work.

1. Suppose $f\colon \mathbf{R}\to\mathbf{R}$ is a function and let $F\colon \mathbf{R}\to\mathbf{R}^2$ be the function defined by $F(x) = [x, f(x)]$. Prove that F is injective, but not surjective. Find a one-sided inverse for F (with proof). Show that it is not unique.

   Note: The image of F is known as the graph of $y = f(x)$.

2. Let $f\colon X\to Y$ be a function and for $x, x' \in X$ define $x \sim x' \Leftrightarrow f(x) = f(x')$. Prove $\sim$ is an equivalence relation on X. Describe the equivalence classes for the case when f is injective. Same for when f is constant. Same for $f\colon \mathbf{R}\to\mathbf{R}$ given by $f(x) = x^2$.

3. Prove that a nonempty finite linearly ordered set has a minimum.

   Hint: Induction on the size of the set.

4. Let $S = \{x \in \mathbf{Q}\colon (\exists n \in \mathbf{Z})[x = 2^n]\}$. If they exist, what are $\max S$ and $\min S$? For S as a subset of $\mathbf{Q}$, same question for $\sup S$ and $\inf S$. Prove your assertions about min and inf.

5. Show that a union of initial segments in $\mathbf{Q}$ is an initial segment. Give a concrete example of a collection of Dedekind cuts, whose union is not a Dedekind cut.

1	2	3	4	5	total (50)	%

Prelim. course grade: %