Midterm 2 / 2012.4.30 / MAT 3013.002 / Foundations of Mathematics

Name: ____

Please show all work.

1. Suppose $f\colon X \to Y$ is a function and $A, B \subseteq X$. Prove that $f_*(A) \setminus f_*(B) \subseteq f_*(A \setminus B)$. Provide a concrete counter example illustrating why containment the other way fails.
2. Suppose $f\colon X \to Y$ is a function and $C, D \subseteq Y$. Prove that $C \subseteq D \Rightarrow f^*(C) \subseteq f^*(D)$. Provide a concrete counter example illustrating why the converse fails.
3. Prove that if $g \circ f$ is 1-1, then f is 1-1. Provide a concrete counterexample of how g need not be 1-1, even if $g \circ f$ is the identity function — make sure you specify the domains and co-domains.
4. Define two points in $\mathbf{R}^2 \setminus \{[0,0]\}$ to be related when one is a nonzero scalar multiple of the other. In other words $[x,y] \sim [x',y'] \Leftrightarrow (\exists c \in \mathbf{R} \setminus \{0\})[[x,y] = [cx', cy']]$. Prove that this is an equivalence relation and describe geometrically the equivalence classes. Sketch.
5. Let $S = \{x \in \mathbf{Q}\colon (\exists n \in \mathbf{N})[x = (-1)^n n/(n+1)]\}$. If they exist, what are the least and greatest elements of S. For S as a subset of $\mathbf{Q}$, same question for $\sup S$ and $\inf S$. Prove your assertions about the least element and the infimum.
6. Prove that if m is the greatest element of a partially ordered set A, then $m = \sup A$.

1	2	3	4	5	6	total (60)	%

Prelim. course grade: %