

Name: _____

Please show all work. Check your answers! 😊

1. Let $A = \begin{bmatrix} 21 & 28 \\ -15 & -20 \end{bmatrix}$ and $b = \begin{bmatrix} 7 \\ -5 \end{bmatrix}$.

- (a) Use Gauss-Jordan elimination to compute the reduced row echelon form of the augmented matrix $[A|b]$ and find all solutions to $Ax = b$. Show steps.
- (b) Can you expect some solutions to $Ax = b$ for arbitrary b ? Explain.

2. Assume A and b are as in the above problem.

- (a) Express $\ker(A)$ as a span of the least number of vectors by solving $Ax = 0$.
- (b) Describe $\ker(A)$ and the solution set to $Ax = b$. Sketch both on the same set of axes.
- (c) Express the image of $x \mapsto Ax$ as a span of the least number of vectors. Describe and sketch the image.

3. Let $u = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

Suppose L is a line through the origin in \mathbf{R}^2 and $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is the reflection with respect to L . Suppose $T(u) = v$.

- (a) Sketch u and v . Then sketch L on the same set of axes.
- (b) Find a unit vector \hat{u} such that $L = \text{span}(\hat{u})$.
- (c) Find a matrix A such that for all vectors x in \mathbf{R}^2 we have $T(x) = Ax$.

4. Let $u = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$, $c = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$.

Suppose $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is a linear map and we know that $T(u) = b$ and $T(v) = c$. Find a matrix A such that for all vectors x in \mathbf{R}^2 we have $T(x) = Ax$.

1	2	3	4	total (40)