Midterm 3 / 2016.12.2 / MAT 2233.002 / Linear Algebra

Name: ______________________________

Please show all work.

1. Let P be the plane $x + 2y + 3z = 0$ in $\mathbf{R}^3$.
   (a) Explain why P is a subspace of $\mathbf{R}^3$.
   (b) Find an orthonormal basis for P.

2. Find an orthonormal basis for the vector space of all real polynomials of degree ≤ 1 with respect to the inner product $\langle p(t), q(t)\rangle = \int_0^1 p(t)q(t)\,dt$.

3. Let $A = \begin{bmatrix} 2 & 3 & 0 & 2 \\ 4 & 3 & 2 & 1 \\ 6 & 5 & 0 & 3 \\ 7 & 0 & 0 & 4 \end{bmatrix}$ and define $T\colon \mathbf{R}^4 \to \mathbf{R}^4$ by $T(\mathbf{x}) = A\mathbf{x}$.
   (a) Compute the determinant of A (show work).
   (b) Extra credit: What can you conclude about T from your answer to part (a)?

4. Let $A = \begin{bmatrix} 2 & 4 \\ 1 & -1 \end{bmatrix}$.
   (a) Find the eigenvalues of A and corresponding eigenvectors. Let S be the matrix whose columns are eigenvectors of A. Compute AS. Verify that $S^{-1}AS$ is diagonal with entries the eigenvalues of A.
   (b) Sketch the eigenspaces and give a geometrical description of the linear map $\mathbf{x} \mapsto A\mathbf{x}$.

1	2	3	4	total (40)