Midterm 3 / 2016.4.29 / MAT 2233.002 / Linear Algebra

Name: ____

Please show all work.

1. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ -3 \\ 3 \end{bmatrix}$. Find the least squares solution x^* to $Ax = b$ and verify that $b - Ax^*$ is orthogonal to the image of A.

2. Let P_3 be the space of all real polynomials $p(t)$ with degree ≤ 3 and let $T\colon P_3 \to P_3$ be the linear map given by $T(p) = tp'' + p$. Find $\det T$.

3. Let $A = \begin{bmatrix} -7 & 3 \\ -18 & 8 \end{bmatrix}$.

   (a) Find the eigenvalues of A and corresponding eigenvectors.

   (b) Let S be the matrix whose columns are eigenvectors of A. Verify that $S^{-1}AS$ is diagonal with entries the eigenvalues of A.

   (c) Sketch the eigenspaces and give a geometrical description of the linear map $\mathbf{x} \mapsto A\mathbf{x}$.

4. Suppose A is an $n \times n$ matrix with n linearly independent eigenvectors corresponding to eigenvalues $\lambda_k (1 \leq k \leq n)$. Find $\det A$ and prove your assertion.

1	2	3	4	total (40)