Name: _____

Please show all work and justify your answers.

1. (10 pts.)   Determine whether the following are vector subspaces of the domain of the linear map $x \mapsto Ax$. If yes, explain. If no, provide a concrete counterexample.

   (a) The solution set of $Ax = b$
   (b) The kernel of A

2. (10 pts.) Determine whether the matrices $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ and $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ are similar. Explain.

3. (10 pts.) Let L be the plane $x_1 + x_2 + x_3 = 0$ and let $v = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

   Define a linear transformation T of L by $Tx = x \times v$. What is the determinant of T?

4. (10 pts.) Find all eigenvalues and the corresponding eigenvectors of

   (a) $A = \begin{bmatrix} 1 & 4 \\ 2 & -1 \end{bmatrix}$

   (b) Reflection of the plane with respect to the main diagonal.

5. (10 pts.) Suppose $A = \begin{bmatrix} 3 & 2 \\ 6 & -1 \end{bmatrix}$. It can be shown that A has eigenvalues

   -3 and 5 with corresponding eigenvectors $\begin{bmatrix} 1 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

   Solve the dynamical system $x(n+1) = Ax(n)$ with initial condition $x(0) = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$.

1	2	3	4	5	total (50)
					%

Prelim. course grade: %