Linear Algebra / MAT 2233
Final / December 16, 1999 / Instructor: D. Gokhman

Name: ______________________ Pseudonym: ______________________

Please show all work and box the answers, where appropriate.

1. (10 pts.) Take an augmented matrix $A = \begin{bmatrix} 2 & 2 & 4 & 0 \\ 1 & 1 & -2 & 2 \end{bmatrix}$.
   (a) Use the row reduction algorithm to bring A to reduced echelon form.
   (b) Find all solutions of the corresponding system. Sketch and describe the solution set.

2. (10 pts.) Determine whether each sequence of vectors is linearly independent. Show work or explain.

   (a) $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ -6 \\ -7 \end{bmatrix}$

   (e) $\left\{(t+1)^2, t^2+1, t^2\right\}$

3. (10 pts.) Suppose $f\colon \mathbf{R}^2 \to \mathbf{R}^2$ is linear, $f\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $f\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$.
   (a) Find the matrix that represents f with respect to the standard basis.
   (b) Is f 1-1? Is f onto? Explain.

4. (10 pts.) Suppose A, B, I are $n \times n$ invertible matrices and I is the identity matrix. Solve matrix equation $A(X+I)A^{-1} = B$ for the $n \times n$ matrix X. Simplify.

5. (10 pts.) Determine whether $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$ is invertible, and if so, find the inverse.

6. (10 pts.) Determine whether each of the following subsets is a vector subspace. If yes, explain. If not, give an explicit example of how an axiom fails.

   (a) $\left\{\begin{bmatrix} a \\ b \end{bmatrix} : a+b+1=0\right\}$ (b) $\left\{\begin{bmatrix} s+1 \\ t-1 \end{bmatrix} : s, t \text{ in } \mathbf{R}\right\}$ (c) $\left\{\begin{bmatrix} t-2s \\ 2s-t \end{bmatrix} : s, t \geq 0\right\}$

   (d) $\{p(t)\colon p(1)=0\}$ in $\mathbf{P}_3$ (e) $\{p(t)\colon p(1)=p(0)\}$ in $\mathbf{P}_3$

7. (10 pts.) Find a basis for: (a) null $\begin{bmatrix} 1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$, (b) span of the sequence in #2d. Sketch and describe both spaces.

8. (10 pts.) Let $A = \begin{bmatrix} 4 & 3 \\ 3 & -4 \end{bmatrix}$. Find the characteristic polynomial, eigenvalues and eigenvectors of A. Find an invertible 2×2 matrix P and a diagonal 2×2 matrix D such that $A = PDP^{-1}$. Compute P^{-1} and verify that $A = PDP^{-1}$.

1	2	3	4	5	6	7	8	total (80)