Linear Algebra / MAT 2233.001
Midterm 2 / April 28, 1999 / Instructor: D. Gokhman

Name: ______________________________

Please show all work.

1. (10 pts.) Suppose A, B, C are invertible $n \times n$ matrices.
   Find an $n \times n$ matrix X such that $A(B + X)C = I$.

2. (20 pts.) Let $A = \begin{bmatrix} 3 & 4 & 2 & 3 \\ -1 & 0 & -2 & -1 \\ 5 & 4 & 6 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

   It can be checked that A is row equivalent to B.
   Find bases for $\operatorname{nul} A$ and $\operatorname{col} A$. Describe and sketch $\operatorname{col} A$.

3. (30 pts.) Determine whether each of the following sets is a subspace of Euclidean space and give reasons for your answer. Sketch each set.

   (a) $\left\{ \begin{bmatrix} s+t \\ t \end{bmatrix} : s, t \text{ in } \mathbf{R} \right\}$ (b) $\left\{ \begin{bmatrix} s+1 \\ t \\ s+t \end{bmatrix} : s, t \text{ in } \mathbf{R} \right\}$ (c) $\left\{ \begin{bmatrix} s+1 \\ t-1 \\ s+t \end{bmatrix} : s, t \text{ in } \mathbf{R} \right\}$

4. (20 pts.) Find $[v]_{\mathcal{B}}$, where

   (a) $v = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ in $\mathbf{R}^2$ and $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$.

   (b) $v = 3 - t^2$ in P_2 and $\mathcal{B} = \{1 + t, 1 - t, 1 + t^2\}$.

1	2	3	4	total (80)	%