Linear Algebra / MAT 2233.001
Final / May 14, 1999 / Instructor: D. Gokhman

Name: ______________________________

Please show all work.

1. (10 pts.) Describe and sketch the general solution of the system of linear equations given by the augmented matrix $\begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. Is the solution a subspace of $\mathbf{R}^3$? Explain.

2. (15 pts.) For each of the following matrices describe and sketch the column space. What is the rank of each matrix?

$$\text{(a)} \begin{bmatrix} 1 & 1 \\ 2 & -2 \end{bmatrix} \qquad \text{(b)} \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} \qquad \text{(c)} \begin{bmatrix} 1 & 0 \\ 2 & 2 \\ 0 & 1 \end{bmatrix}$$

3. (15 pts.) For each of the matrices in the preceding problem consider the corresponding linear map T. In each case, what are the dimensions of the kernel and the range of T? Is T 1-1? Onto? Explain.

4. (15 pts.) Find the standard matrix for each linear map $T\colon \mathbf{R}^n \to \mathbf{R}^n$, where
   (a) $n = 2$ and T is the rotation by $3\pi/2$.
   (b) $n = 3$ and T is the rotation by π with respect to the x_1-axis.
   (c) $n = 3$ and T is the reflection with respect to the plane $x_2 = 0$.

5. (10 pts.) For which λ is the sequence $\begin{bmatrix} 11-\lambda \\ -18 \end{bmatrix}, \begin{bmatrix} 6 \\ -10-\lambda \end{bmatrix}$ not linearly independent? How does the dimension of the span of this sequence depend on λ?

6. (15 pts.) Suppose A, B, C are invertible $n \times n$ matrices.
   Solve the following equations for an $n \times n$ matrix X. Simplify.

$$\text{(a) } B^{-1}XB = C \qquad \text{(b) } XAB + I = C \qquad \text{(c) } ABCXABC = I$$

7. (10 pts.) Let $A = \begin{bmatrix} 1 & 2 & 2 & 4 \\ 3 & 6 & 0 & 6 \\ 5 & 10 & 4 & 14 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.
   It can be checked that A is row equivalent to B. Find bases for $\operatorname{nul} A$ and $\operatorname{col} A$.

8. (10 pts.) Find $[v]_{\mathscr{B}}$, where
   (a) $v = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ in $\mathbf{R}^2$ and $\mathscr{B} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$.
   (b) $v = 2 - 3t$ in P_1 and $\mathscr{B} = \{2+t, 2-t\}$.

1	2	3	4	5	6	7	8	total (100)	%

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