Final exam / 2007.5.10 / Calculus III / MAT 2213.003

Name: ______________________________

Please show all work and justify your answers. Supply brief narration with your solutions and draw conclusions.

1. Evaluate the following infinite sums

$$\text{(a)} \sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{3^n} \qquad \text{(b)} \sum_{n=3}^{\infty} \frac{1}{n^2+n}$$

2. Determine whether the following series converge

$$\text{(a)} \sum_{n=1}^{\infty} \frac{\ln n}{n^2} \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{\sin^2 n}{1+n^2}$$

3. Find a power series representation for $\dfrac{1}{8-x^3}$ and determine its interval of convergence.

4. Find an equation for the plane containing the line $2t\,\hat{\imath}+(1+t)\,\hat{\jmath}+(1-t)\,\widehat{k}$ and the point $2\hat{\imath}+\widehat{k}$. Sketch.

5. Find a parametric formula for the line tangent to the curve $t\,\hat{\imath}+t^2\,\hat{\jmath}+t^3\,\widehat{k}$ at the point $\hat{\imath}+\hat{\jmath}+\widehat{k}$.

   Hint: First find t which gives you the point.

6. Consider the surface given by $z = y\cos(xy)$.

   (a) Find the differential. In other words, express dz in terms of dx and dy.

   (b) Find an equation for the plane tangent to the surface at the point $\hat{\jmath}+\widehat{k}$.

7. Suppose f is a differentiable function of x and y and $g(r,\theta) = f(r\sin\theta, r\cos\theta)$.

   (a) Use the table of values below to find the gradient of g at the point $\hat{\imath}$.

   (b) What is the maximum possible value of the directional derivative of g at $\hat{\imath}$ along the various directions and along which direction is this maximum attained?

(x,y)	f	g	f_x	f_y
(0,1)	2	3	4	5
(1,0)	6	7	8	9

8. Integrate $x\cos(x-y)$ over the rectangle $[-\frac{\pi}{3}, \frac{\pi}{3}] \times [0, \frac{\pi}{6}]$.

1	2	3	4	5	6	7	8	total (80)