

Name: \_\_\_\_\_

1. (30 pts.) CONIC SECTIONS

For each of the following conic sections, write an equation describing the curve and find the eccentricity.

- (a) a circle passing through the point  $(2, -4)$  with center  $(4, 4)$ .
- (b) a hyperbola with foci  $(\pm 2, 0)$  and asymptotes  $y = \pm 2x$ .
- (c) a parabola with vertex  $(4, -4)$  and focus  $(8, -4)$ .

(Hint: find the directrix and then another point on the parabola)

2. (20 pts.) CALCULUS ON CURVES

Suppose the position of a fly on a window is given as a function of time  $t$  by  $x = \sec t, y = \tan t$ . At time  $t$  find the velocity components  $\frac{dx}{dt}, \frac{dy}{dt}$ , the slope  $\frac{dy}{dx}$ , and the curvature of the path  $\frac{d^2y}{dx^2}$ . At time  $t = \frac{\pi}{6}$  find an equation for the tangent line.

3. (20 pts.) ROTATION

How are the following equations transformed by a rotation by  $\frac{\pi}{4}$ ?

(a)  $y = x + 1$       (b)  $y = x^2$

4. (30 pts.) CURVES IN POLAR COORDINATES

- (a) Sketch the curve  $r = 2 \sin(4\theta)$ . Find the area enclosed by one of the petals.
- (b) Find all points of intersection of the curves  $r = 1$  and  $r = 2 \sin(2\theta)$ .
- (c) Find the length of the curve  $r = 2 \sin^3\left(\frac{\theta}{3}\right), \theta \in [0, 3\pi]$ .

1	2	3	4	total (100)