Calculus III, MAT 2213
Exam №1, 3/3/93
Instructor: D. Gokhman

Name: _____

1. (20 pts.) Determine whether each of the following sequences converges to a real number. If so, find the limit. Otherwise state that the sequence diverges.

   (a) $\ln(n) - \ln(n+15)$ (b) $\cos\left(\frac{n\pi}{2}\right)$ (c) $\sqrt[n]{5^{2n+1}n}$ (d) $\left(\frac{n-1}{n+1}\right)^n$

2. (20 pts.) Determine whether each of the following series converges to a real number. Otherwise state that the series diverges.

   (a) $\sum_{n=0}^{\infty}(-1)^n$ (b) $\sum_{n=0}^{\infty}\frac{(n!)^2}{(2n)!}$ (c) $\sum_{n=2}^{\infty}\frac{1}{n\log(n)}$ (d) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n^2+1}}$

3. (20 pts.) Find the interval of convergence for the following series:

   (a) $\sum_{n=0}^{\infty}\frac{(x+1)^n}{n}$ (b) $\sum_{n=1}^{\infty}\frac{x^n}{n^n}$

4. (40 pts.) Find the Taylor series with center c for the following functions $f(x)$.

   (a) $f(x) = x^5e^x, \quad c = 0$ (b) $f(x) = \frac{1}{\sqrt{1-x^2}}, \quad c = 0$

   (c) $f(x) = \frac{1}{x}, \quad c = 1$ (d) $f(x) = \ln(x), \quad c = 1$