

Name: _____

Please show all work.

1. Find all simultaneous solutions to the system of equations

$$3x \equiv 4 \pmod{8}$$

$$3x \equiv 8 \pmod{7}$$

2. Fibonacci numbers f_n ($n \geq 0$) are defined recursively by $f_0 = 0, f_1 = 1$ and for $n > 1$

$$f_n = f_{n-1} + f_{n-2}$$

- (a) Compute the Fibonacci numbers for $n \leq 10$
(b) Prove that $f_n < 2^n$ for all $n \geq 0$

3. Let $z = \frac{1 + 5i}{1 + i\sqrt{2}}$

- (a) Simplify z
(b) Find the real and imaginary parts of z
(c) Sketch z in the complex plane.
(d) Find $|z|$

Hint: to save work, first find $|1 + 5i|$ and $|1 + i\sqrt{2}|$

| 1 | 2 | 3 | total (30) |
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