

Name: \_\_\_\_\_

Please show all work.

1. (a) If  $P, Q, R$  are propositions, use a truth table to prove that  

$$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$$
 (b) If  $X, Y, Z$  are sets, prove that  $X \cap (Y \cup Z) = (X \cap Y) \cup (X \cap Z)$
2. Consider the Diophantine equation  $15x - 24y = 9$ 
  - (a) Find the general integer solution to the equation.
  - (b) Find three distinct particular integer solutions to the equation and sketch them in the plane.
3. Find all simultaneous integer solutions to the system of equations

$$2x \equiv 4 \pmod{9}$$

$$3x \equiv 8 \pmod{11}$$

4. A sequence  $a_n \in \mathbf{Z}, n \geq 0$  is defined recursively by  $a_0 = 3, a_1 = 10$  and for  $a > 1$

$$a_n = 7a_{n-1} - 12a_{n-2}$$

- (a) Compute  $a_n$  for  $n \leq 6$
  - (b) Prove by induction that  $a_n = 2 \cdot 3^n + 4^n$  for all  $n \geq 0$
5. Let  $p \in \mathbf{Z}[x], p(x) = x^4 - x^3 + x - 1$ . By inspection  $p(1) = 0$ . Use this to find all complex roots of  $p$  and sketch them in the complex plane.

1	2	3	4	5	total (50)