Midterm 2 / 2010.11.29 / MAT 1224.008 / Calculus II

Name: _____

Please show all work and justify your answers.

1. Use trigonometric substitution to evaluate

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

2. Find all solutions to the following equations for $y$ as a function of $x$.

$$\text{(a) } 2\sqrt{xy}\,\frac{dy}{dx} = 1, \quad x, y > 0 \qquad \text{(b) } \frac{dy}{dx} - xy = x, \quad y(0) = 3$$

3. Evaluate the following sums

$$\text{(a) } \sum_{n=1}^{\infty} \left[\frac{5}{2^n} + \frac{1}{3^n}\right] \qquad \text{(b) } \sum_{n=1}^{\infty} nx^n$$

[Hint for (b): recognize the series as $x$ times the derivative of a known series]

4. Find Taylor series at $x = c$ and determine the interval of convergence. If you have trouble with writing out the general series, compute the first four nonzero terms for partial credit.

$$\text{(a) } \frac{x^{77}}{2-x}, \quad c = 0 \qquad \text{(b) } \ln x, \quad c = 1$$

[Hint for (a): You don't want to use Taylor's formula alone, trust me]

5. Find the first five nonzero terms of the Fourier series for the function on the interval $[-2, 2]$ defined by $f(x) = x^2$ for $x$ between $-1$ and $1$ and $f(x) = 0$ otherwise.

| 1 | 2 | 3 | 4 | 5 | total (50) | % |
|---|---|---|---|---|---|---|
| | | | | | | |

Prelim. course grade: %