Midterm 2 / 2006.4.27 / Calculus II / MAT 1223.005

Name: _____

Please show all work and justify your answers. Make and label sketches. Supply brief narration with your solutions and draw conclusions, including units as appropriate.

1. A conical tank, with vertex at the bottom, height 5 m and top diameter 4 m is full of water. How much work is involved in pumping all the water over the rim? Feel free to integrate numerically.

   Note: mass density of water $\delta = 10^3$ kg/m^3, gravitational acceleration $g = 9.81$ m/s^2

2. Find the second degree Taylor approximation to $e^x(1-x)$ near $x = 0$. Sketch the given function and the approximation fairly close to $x = 0$ on the same graph. On which interval would you say the approximation is "good"?

3. Find the first order Fourier approximation to $f(x) = \frac{1}{2} - |x|$ on the interval $[-1, 1]$. Feel free to compute the required integrals numerically. Sketch $f(x)$ and the approximation over the entire interval on the same graph. At what points is the approximation the poorest? Why do you think that is?

4. Suppose $y(x)$ is a solution of the differential equation

$$\frac{dy}{dx} = (x+1)(y^2+1)$$

   satisfying the initial condition $y(0) = 1$. Find $y(0.5)$.

Fourier series: If f is a continuous function on $(-p/2, p/2)$, then $f(x) = a_0 + \sum_{k=1}^{\infty}[a_k \cos(2\pi kx/p) + b_k \sin(2\pi kx/p)]$, where $a_0 = \frac{1}{p}\int_{-\frac{p}{2}}^{\frac{p}{2}} f(x)\,dx$, $a_k = \frac{2}{p}\int_{-\frac{p}{2}}^{\frac{p}{2}} f(x)\cos(2\pi kx/p)\,dx$, $b_k = \frac{2}{p}\int_{-\frac{p}{2}}^{\frac{p}{2}} f(x)\sin(2\pi kx/p)\,dx$

1	2	3	4	total (40)	%

Prelim. course grade: %