Name: _____

Please show all work and justify your answers. Make and label sketches. Supply brief narration with your solutions and draw conclusions, including units as appropriate.

1. Find the area ouside the circle $r = 1/2$ and inside the cardioid $r = 1 - \sin\theta$. Sketch.

   [Hint: Find the polar coordinates of the points of intersection. You may use the calculator to evaluate the integral.]

2. Find the trapezoidal and midpoint approximations with 2 subdivisions to the integral of $x^2$ from 1 to 2. Are these over or under estimates? Explain. Sketch.

3. Determine whether the improper integral $\displaystyle\int_0^1 \frac{\sqrt{\sqrt{x}+1}}{\sqrt[3]{x^4}}\,dx$ converges. Justify.

4. A small barn, the shape of a hemisphere of radius 10 m, is filled with hay. The density of hay decreases linearly with height from 200 kg/m$^3$ at the bottom to 100 kg/m$^3$ at the top. How high above the ground is the center of mass of the hay? Sketch.

   [Hint: Write down a formula for the density as a function of height. Set up integrals for the total mass of hay and for the vertical moment. You may evaluate integrals on the calculator.]

5. Demonstrate your mastery of techniques of integration by evaluating the following antiderivatives manually. Show all steps.

$$\text{(a)} \int x^2\sqrt{x+1}\,dx \qquad \text{(b)} \int \frac{x^2}{x^2-1}\,dx$$

| 1 | 2 | 3 | 4 | 5 | total (50) | % |
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Prelim. course grade:         %