Final exam / 2003.5.5 / Calculus II / MAT 1223.002

Name: ______________________________

Please show all work and justify your answers.

1. (10 pts.) A valve for an internal combustion engine is a solid obtained by rotating around the y axis the region in the positive quadrant of the plane bounded by the lines $y = 5$ and $y = \frac{1}{x} - 1$. Sketch the valve. Assuming uniform mass density, find the valve's center of mass. You may evaluate integrals numerically.

2. (10 pts.) Determine whether the following improper integrals converge. Justify.

$$\text{(a)} \int_0^1 \left(\frac{\cos x}{x}\right)^{\frac{2}{3}} dx \quad \text{(b)} \int_2^\infty \frac{\sqrt{x+1}}{\sqrt[3]{x^2-1}}\, dx$$

3. (10 pts.) Christie Brinkley approximates an integral using the trapezoidal rule with 5 subdivisions. Chuck Norris is beefier and makes 10 subdivisions. Based on their approximations 18.243 (Christie) and 18.232 (Chuck) estimate the exact value of the integral.

4. (20 pts.) Demonstrate your mastery of techniques of integration (other than guess-and-check) by evaluating the following integrals. Show all work. Name the techniques you are using. If you use tabulated integrals, cite them.

$$\text{(a)} \int \frac{x^2}{x+1}\, dx \quad \text{(b)} \int \frac{1}{x^2+6x+13}\, dx \quad \text{(c)} \int \frac{\sqrt{1-\ln x}}{x}\, dx \quad \text{(d)} \int \ln x\, dx$$

5. (10 pts.) The likely duration (in minutes) of an internet surfing session is modeled by a decaying exponential probability density ke^{-rt}, where k and r are positive constants.

   (a) Express the proportionality constant k in terms of r.
   (b) To estimate r, a timing experiment is performed with many surfers. What should the value of r be, if half the surfers are finished after 20 minutes?
   (c) What is the average duration of a surfing session predicted by this model?

6. (10 pts.) Find the second order Taylor approximation to $\cos x / \sqrt[3]{1+x}$ near $x = 0$. Sketch the given function and the approximation very close to $x = 0$ on the same graph.

7. (10 pts.) Find the first order Fourier approximation on the interval $[-4, 4]$ to the signal $f(x) = 1$, if $-1 \le x \le 3$ and $f(x) = 0$ otherwise. You may evaluate integrals numerically. Sketch $f(x)$ and the approximation over the entire interval on the same graph.

8. (20 pts.) Bill Bennett tries out a new gambling strategy, where his tally $y(t)$ (in thousands of dollars) as function of time spent in the casino (in hours) is governed by the differential equation $y'(t) = y(t+1)^{-2}$. Having walked in with 5 thousand dollars, Bill would like to determine how much he will have after 1 hour of gaming.

   (a) Estimate $y(1)$ using Euler's method with step size $\Delta t = 0.5$.
   (b) Solve the differential equation analytically and find $y(1)$.
   (c) Sketch $y(t)$ over an extended stay at the casino. Should Bill keep playing?

1	2	3	4	5	6	7	8	total (100)