Calculus II / MAT 1223
Quizzes / Fall 1999 / Instructor: D. Gokhman

1. Evaluate:

$$\text{(a) } \int \frac{\left(z^3+1\right)^2}{\sqrt{z}}\,dz \qquad \text{(b) } \int_1^2 \left(z^2+1\right)^2\,dz \qquad \text{(c) } \int \frac{\sqrt[3]{\sqrt{t}+1}}{\sqrt{t}}\,dt$$

$$\text{(d) } \frac{d}{dx}\int_x^0 \sin(t)\,dt \qquad \text{(extra credit) } \frac{d}{dx}\int_0^{x^2} \sin(t)\,dt$$

2. Sketch the curves $4y^2 - 2x = 0$ and $4y^2 + 4x - 12 = 0$ and find the area between them.

3. Find the volumes of solids of revolution generated by rotating the given region with respect to the specified axis. Sketch the region, the "rectangles", and the axis.
   (a) Region bounded by $y - \sqrt[3]{x} = 0$, $x = 8$, $y = 0$. Axis: $x = 0$.
   (b) Region bounded by $x^2 + y^2 = 1$, $x = 0$, $y = 0$ ($x \geq 0$, $y \geq 0$). Axis: $y = 0$.

4. (a) Find the length of the curve given by $y = \left(1 - x^{\frac{2}{3}}\right)^{\frac{3}{2}}$, $0 \leq x \leq 1$.
   (b) Find the surface area generated by rotating the curve $x = 1 - t^2$, $y = 2t$, $0 \leq t \leq 1$ around the x axis.

5. (a) Given a linear spring with Hooke's constant $k = 2$ kg/s^2, find the work needed to compress the spring 1 meter from equilibrium.
   (b) Find the centroid of the region between the curves $y = x^2$ and $y = x + 2$. Sketch.

6. (a) Differentiate $\ln(x)\ln(x^2+1)$.
   (b) Find y' if $y = x^x$. (Hint: take ln of both sides and differentiate implicitly)
   (c) Evaluate $\displaystyle\int \frac{x\,dx}{x^2+1}$.
   (d) Sketch $y = \ln(1/x)$. (Hint: simplify first)

7. (a) Differentiate $e^{x\ln x}$.
   (b) Evaluate $\displaystyle\int e^{x+e^x}\,dx$.
   (c) Let R be the region in the plane bounded by $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Find the volume of the solid formed by rotating R around the y axis. Sketch.

8. (a) Differentiate: (i) $\log_5(x^2+1)$, (ii) 5^{x^2+1},
   (b) Evaluate: (i) $\displaystyle\int 5^{2x}\,dx$, (ii) $\displaystyle\int \sqrt{x}\,2^{x^{3/2}}\,dx$,
   (c) Sketch $y = \tan^{-1}x$ and find $\displaystyle\lim_{x\to\infty}\tan^{-1}x$ and $\displaystyle\lim_{x\to-\infty}\tan^{-1}x$.

9. (a) Differentiate: (i) $x\tan^{-1}(x)$, (ii) $\sin^{-1}(2x^2)$, (iii) $\sinh(x)\cosh(2x)$.
   (b) Evaluate integrals: (a) $\displaystyle\int \tan(x)\,dx$, (b) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$.
   (c) Sketch $\sinh^{-1}x$ and find $\displaystyle\lim_{x\to\infty}\sinh^{-1}x$.

10. Evaluate integrals: (a) $\displaystyle\int \frac{\sin x - \cos x}{\sin x}\,dx$, (b) $\displaystyle\int x\sqrt{x-1}\,dx$, (c) $\displaystyle\int \frac{dx}{\sqrt{4x-x^2}}$.