Name: _____

Please show all work and justify your statements. Make and label sketches, draw conclusions (using complete sentences and including units), and box the final answers as appropriate.

1. Let $f(x) = x^3 - x^2$. Find all critical points and classify them as local minima or maxima. Find all inflection points. Specify the intervals of which f is increasing versus decreasing and concave up versus down. Sketch.

2. Estimate how accurately the radius of a 10 m high cylindrical tank must be measured to calculate the tank's volume within 1% of its true value.

3. Suppose the rate of growth of a mycelium is modelled by $10 - 2^{-t}$ kg/year, where t is time (yrs). Find upper and lower estimates for the change in size over the period of 2 years using 3 subintervals of equal duration starting from $t = 0$. Use the Fundamental Theorem of Calculus to obtain an exact value for the integral and compare it to the estimates you obtained, what do you observe? If the mycelium is starts at 80 kg at $t = 0$, what is its size in 2 years?

4. Evaluate the following definite integrals analytically. Show all steps.

$$\text{(a) } \int_1^8 t^3 \sqrt[3]{t}\, dt \qquad \text{(b) } \int_{-\frac{\pi}{2}}^{\pi} \sin(6t)\, dt$$

5. Evaluate the following antiderivatives analytically. Show all steps.

$$\text{(a) } \int \frac{1+t}{t}\, dt \qquad \text{(b) } \int 5^{2t}\, dt$$

1	2	3	4	5	total (50)