Final exam / 2002.5.7 / Calculus I (honors) / MAT 1214.001

Name: ______________________________

Please show all work and justify your answers.

1. (10 pts.) Suppose $f(x) = \begin{cases} x & \text{for } x < 1 \\ 2-x & \text{for } x > 1 \\ 0 & \text{for } x = 1 \end{cases}$.
   (a) What is $\lim_{x\to 1} f(x)$?
   (b) Explain why this limit exists.

2. (10 pts.) Evaluate $\lim_{x\to 0} \dfrac{x\tan(2x)}{\sin^2(3x)}$.

3. (10 pts.) Assuming a yearly inflation rate of 17% the price of a liter of milk is given by $P(t) = 0.5\cdot 1.17^t$ where t is in years. How fast will the price of milk be rising in 15 years?

4. (10 pts.) Let $f(x) = x^{\frac{1}{3}}$.
   (a) Use the definition of derivative to find f' and show that it satisfies the power rule. Hint: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$.
   (b) Find an equation for the tangent line to f at $x = 64$ and use it to approximate $62^{\frac{1}{3}}$.

5. (10 pts.) Find dy/dx, if $x\cos(y) + y^2 = \ln(x-y) + 1$.

6. (10 pts.) HMS Rustbucket breaks up and starts dumping its cargo of crude oil at the rate of 80 liters per minute, forming a circular oil slick 0.2 mm thick. When the oil slick is 4 km in diameter how fast is the diameter increasing?

7. (10 pts.) An artillery position on a hill 120 meters above the plain fires a missile. The horizontal distance of the missile from the firing point and its height above the plain are given (in meters) as functions of time (in seconds) by $x(t) = 80t$, $y(t) = 120 + 10t - gt^2/2$, where $g = 9.81$ m/sec^2.
   (a) When does the missile's trajectory reach its highest point?
   (b) When does the missile hit the plain?
   (c) What is the speed of the missile when it hits the plain?

8. (10 pts.) Sue wants to build a 2 m^2 flower bed in the shape of a pizza slice (sector) by placing bricks along the border. What angle and radius should she choose in order to minimize the cost of bricks?

9. (10 pts.) Evaluate the following integrals

   (a) $\displaystyle\int_1^4 \left[\sqrt{t} + t^3\right]^2 dt$ (b) $\displaystyle\int_0^{\frac{\pi}{8}} \cos(4t)\,dt$ (c) $\displaystyle\int \frac{1+t^2}{t^3}\,dt$ (d) $\displaystyle\int 2^{3t}\,dt$

10. (10 pts.) Cholesterol starts depositing on arterial walls at a rate proportional to the cube root of time. After 2 years, the deposit is 0.5 mm thick. How thick is the deposit 15 years after it started forming?

1	2	3	4	5	6	7	8	9	10	total (100)	(%)