

Name: _____

Please show all work and justify your answers.

1. (20 pts.) Find dy , where

(a) $y = \cos^3(\sqrt{1-x^2})$

(b) $(xy^2 + 1)^7(x - y^3)^8 = 1$

2. (20 pts.) Find the linear approximation to $f(x) = \sqrt{1-x^2}$ near $x = -1/2$. Sketch both $y = f(x)$ and the linear approximation.

3. (25 pts.) Agent 007 orders a vodka martini shaken, not stirred. His martini glass is conical, 10 cm in diameter and 7 cm deep. Boris, the bartender, drops a 1 cm^3 green olive into the glass. Boris summons his geometry skills learned at bartending college and computes the exact volume of martini required to fill the glass, taking into account the fact that the olive is already on the bottom of the glass and will remain there throughout the pouring process. Finally Boris starts pouring the martini into the glass at the constant rate of $2 \text{ cm}^3/\text{sec}$.

- (a) What is the total volume of martini computed by Boris?
(the volume of a cone of height h and radius r is $\pi r^2 h/3$)
- (b) How long must 007 wait for Boris to finish pouring?
- (c) Sketch 007's glass with the olive on the bottom and martini level high enough to cover the olive. What is the relationship between the level and volume of martini in 007's glass during the pouring process?
- (d) What is the level of martini in the glass, when Boris is half way through pouring?
- (e) How fast is the martini level rising, when Boris is half way through pouring?

1	2	3	total (65)	(%)