

Name: \_\_\_\_\_ Pseudonym: \_\_\_\_\_

Please show all work and box the answers, where appropriate.

1. (20 pts.) Evaluate each of the following limits ( $\infty$  and  $-\infty$  are legitimate answers) or state that the limit does not exist and *explain*.

$$(a) \lim_{x \rightarrow \infty} \frac{x^2 - 4}{x^3 + 8} \quad (b) \lim_{x \rightarrow 0} \frac{x^5}{|x|^3} \quad (c) \lim_{x \rightarrow 0} \frac{\tan(3x)}{\sin(2x)} \quad (d) \lim_{x \rightarrow 0} \frac{\sin(3x^2)}{\tan(2x^3)}$$

2. (30 pts.) Let  $f(x) = 2/x^2$ .

- (a) Find all asymptotes of each type or state that there are not any of the given type.  
 (b) Find all  $x$ , where  $f$  fails to be continuous.  
 (c) Show that  $f$  is differentiable at  $x = -1$  by computing  $f'(-1)$  from its definition and check your answer by computing  $f'(-1)$  using the rules of differentiation.  
 (d) Find an equation for the line tangent to the graph of  $y = f(x)$  at  $x = -1$ .  
 (e) Sketch the graph of  $y = f(x)$  and of the tangent line.

3. (20 pts.) Find the derivatives of the following functions with respect to  $x$ .

$$(a) x^2 \tan^4(x^3 + 1) \quad (b) \frac{2x + 1}{x^2 + 3}$$

4. (20 pts.)

- (a) Let  $f(x) = \sqrt{2x + 1}$ . Find the differential  $df$ .  
 (b) Suppose  $x^2 y^3 \sin(x^2 y^3) = 1$ . Find  $dy/dx$ .

5. (20 pts.) Consider  $f(x) = 2 \sin x - x$  on the interval  $[-\pi, \pi]$ .

- (a) Find all critical points of  $f$  in the given interval and classify them using the second derivative test.  
 (b) Find the global maximum and global minimum of  $f$  on the given interval and find all  $x$  where they are attained. Sketch the graph of  $y = f(x)$  on the above interval.

6. (20 pts.) Sketch  $y = f(x)$  on the interval  $[-1, 1]$  and determine whether the hypotheses of the Mean Value Theorem are satisfied and explain. If possible, find all points in the interval satisfying the conclusion of the theorem.

$$(a) f(x) = |x|^3 \quad (b) f(x) = \sqrt{|x|}$$

7. (20 pts.) Consider  $f(x) = 3 - |x|$  on the interval  $[-2, 2]$ . Compute the upper and lower Riemann sums with  $n = 3$ . For each sum, sketch the corresponding rectangles on the graph of  $y = f(x)$ . Compare your results to the exact area under the graph of  $y = f(x)$ .

8. (20 pts.) Evaluate

$$(a) \frac{d}{dx} \int_1^{\tan(2x)} \sqrt{1+t^2} dt \quad (b) \int_{\pi}^{3\pi/2} \sin(x) \cos^3(x) dx$$

1	2	3	4	5	6	7	8	total (170)	%