Calculus I, MAT 1214 (1)
Final, May 7, 1996
Instructor: D. Gokhman

Name: _____ Pseudonym: _____

Show work. Answers alone are not sufficient. Box your answers.

1. (40 pts.) For each of the following curves find an equation for the line tangent to the curve at the given point.

   (a) $(x^3+y^3)^2 = 4xy, \quad (1,1)$.
   (b) $\cos(xy) = 1-x-y, \quad (0,0)$.

2. (40 pts.) For each of the following functions f

   (i) find all critical points of f in the given interval,
   (ii) classify these critical points,
   (iii) determine the intervals on which f is increasing or decreasing,
   (iv) determine whether the global maximum and global minimum of f exist and, if so, find their values.
   (v) sketch $y = f(x)$.

   (a) $f(x) = \dfrac{x^2-2x+2}{x-1}, \quad (-\infty, 1)$
   (b) $f(x) = x^{\frac{2}{3}}, \quad (-\infty, \infty)$

3. (60 pts.) Find the following antiderivatives

   (a) $\displaystyle\int \frac{dx}{\sqrt[5]{x^2}}$ (b) $\displaystyle\int x^3\sqrt{5x^4-1}\,dx$ (c) $\displaystyle\int \cos^4(3x)\sin(3x)\,dx$

4. (60 pts.) Evaluate the following integrals

   (a) $\displaystyle\int_{-3}^{1} 3x^2\,dx$ (b) $\displaystyle\int_{-2}^{2} 2|x-1|\,dx$ (c) $\displaystyle\int_{0}^{2} 3\left|x^2-1\right|\,dx$

(CONTINUED ON REVERSE)

1	2	3	4	5	6	(300)	%

5. (40 pts.) Construct the following approximations to the integral in (4a):

   (a) The left endpoint Riemann sum L with $n = 2$.

   (b) The right endpoint Riemann sum R with $n = 2$.

   (c) The midpoint Riemann sum M with $n = 2$.

   (d) The Simpson approximation $S = \dfrac{4M + L + R}{6}$.

6. (60 pts.) True/false questions. Circle your choice. Justification not required.

T F (a) If $f(x)$ is continuous on $(0, 1)$ then it has a maximum on $(0, 1)$.

T F (b) If $\displaystyle\int_0^x f(t)\,dt = \int_0^x g(t)\,dt$ for all x, then $f(x) = g(x)$.

T F (c) If $f'(x) = g'(x)$ for all x, then $f(x) = g(x)$.

T F (d) If f is differentiable, $f'(0) = 0$, $f'(x) < 0$ for $x > 0$, and $f'(x) > 0$ for $x < 0$, then f has a maximum at 0.

T F (e) If f is differentiable, $f(-1) = 1$ and $f(1) = 1$, then f has a critical point.

T F (f) If $-2 \le f(x) \le 2$ for all x, then $-6 \le \displaystyle\int_0^3 f(x)\,dx \le 6$.

HAVE A NICE SUMMER!