Calculus I, MAT 1214 (2–5)
Final, May 11,1995
Instructor: D. Gokhman

Name: ____

Show your reasoning, answers alone are not sufficient. Calculators or similar devices not permitted.

1. (40 pts.) Find the following limits:

$$\text{(a)}\ \lim_{x\to 1}\frac{x^2-1}{x-1} \qquad \text{(b)}\ \lim_{x\to 0^-}\frac{x+2|x|}{x-2|x|} \qquad \text{(c)}\ \lim_{x\to 0}\frac{1-\cos^2 x}{x} \qquad \text{(d)}\ \lim_{x\to 0} x^3\sin\left(\frac{1}{x^5}\right)$$

2. (40 pts.) Differentiate each of the following functions:

$$\text{(a)}\ 5 \qquad \text{(b)}\ \frac{2x^3}{x^4+1} \qquad \text{(c)}\ x^5\cos^5(x^5) \qquad \text{(d)}\ \sqrt[3]{\sin^5 x-2}$$

3. (20 pts.) Use the linear approximation to $\sqrt{x}$ at a suitable point a to approximate $\sqrt{10}$.

4. (20 pts.) Find the minimum and maximum values of $f(x)=\dfrac{1}{x^2+1}$ on the interval $[-1,1]$.

5. (30 pts.) True/false questions. Circle your answer, no justification necessary.

T F (a) If $f'(x)=g'(x)$ for all x, then $f(x)=g(x)$.

T F (b) If $f'(x)$ exists for all real x and $f(0)$ is the maximum of $f(x)$, then $f'(0)=0$.

T F (c) If $f'(x)$ exists for all real x, $f(0)=1$, and $f(1)=-1$, then $f'(a)=-2$ for some a.

T F (d) If $f(x)$ is continuous at all x, then $f(x)$ has a maximum.

T F (e) If $g_1(x)\le f(x)\le g_2(x)$ for all x and g_1, g_2 have limits at $x=0$, then so does f.

T F (f) If $f(x)\ge -5$ for all x, then $\int_0^2 f(x)\,dx\ge -10$.

6. (40 pts.) Find all antiderivatives for each of the following functions:

$$\text{(a)}\ 0 \qquad \text{(b)}\ 5 \qquad \text{(c)}\ x^2\sin(5x^3) \qquad \text{(d)}\ \frac{x}{\sqrt{x^2+1}}$$

7. (30 pts.) Sketch the region bounded by the graph of $y=|x^3|-1$ and the x axis. Find the area of this region.

8. (40 pts.) Let $f(x)=\dfrac{x^2+1}{x+1}$.

   (a) Find $f'(x)$,
   (b) Find $f''(x)$,
   (c) Find all critical points,
   (d) Classify the critical points as local minima, local maxima or neither,
   (e) Specify where the graph is increasing/decreasing,
   (f) Specify where the graph is concave up/down,
   (g) Find equations of all asymptotes,
   (h) Sketch the graph of $y=f(x)$.

1	2	3	4	5	6	7	8	total (260)